This is the March 2011 version of the Grade 3 Model Curriculum for Mathematics. The current focus of this document is to provide instructional strategies and resources, and identify misconceptions and connections related to the clusters and standards. The Ohio Department of Education is working in collaboration with assessment consortia, national professional organizations and other multistate initiatives to develop common content elaborations and learning expectations.

Grade 3	
Domain	**Cluster**
Operations and Algebraic Thinking	• Represent and solve problems involving multiplication and division. • Understand properties of multiplication and the relationship between multiplication and division. • Multiply and divide within 100. • Solve problems involving the four operations, and identify and explain patterns in arithmetic.
Number and Operations in Base Ten	• Use place value understanding and properties of operations to perform multi-digit arithmetic.
Number and Operations—Fractions	• Develop understanding of fractions as numbers.
Measurement and Data	• Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects. • Represent and interpret data. • Geometric measurement: understand concepts of area and relate area to multiplication and to addition. • Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.
Geometry	• Reason with shapes and their attributes.

Grade 3

Domain	**Operations and Algebraic Thinking**
Cluster	***Represent and solve problems involving multiplication and division.***
Standards	1. Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.* 2. Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.* 3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1] 4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = □ ÷ 3, 6 × 6 = ?.*

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

As the framework for the assessments, this section will be developed by the CCSS assessment consortia (SBAC and PARCC). Ohio is currently participating in both consortia and has input into the development of the frameworks. This information will be included as it is developed.

Instructional Strategies and Resources

Instructional Strategies

In Grade 2, students found the total number of objects using rectangular arrays, such as a 5 x 5, and wrote equations to represent the sum. This strategy is a foundation for multiplication because students should make a connection between repeated addition and multiplication.

Students need to experience problem-solving involving equal groups (whole unknown or size of group is unknown) and multiplicative comparison (unknown product, group size unknown or number of groups unknown) as shown in Table 2 of the Common Core State Standards for Mathematics, page 89. No attempt should be made to teach the abstract structure of these problems.

Encourage students to solve these problems in different ways to show the same idea and be able to explain their thinking verbally and in written expression. Allowing students to present several different strategies provides the opportunity for them to compare strategies.

Sets of counters, number lines to skip count and relate to multiplication and arrays/area models will aid students in solving problems involving multiplication and division. Allow students to model problems using these tools. They should represent the model used as a drawing or equation to find the solution.

This shows multiplication using grouping with 3 groups of 5 objects and can be written as 3 × 5.

Provide a variety of contexts and tasks so that students will have more opportunity to develop and use thinking strategies to support and reinforce learning of basic multiplication and division facts.

Have students create multiplication problem situations in which they interpret the product of whole numbers as the total number of objects in a group and write as an expression. Also, have students create division-problem situations in

which they interpret the quotient of whole numbers as the number of shares.

Students can use known multiplication facts to determine the unknown fact in a multiplication or division problem. Have them write a multiplication or division equation and the related multiplication or division equation. For example, to determine the unknown whole number in 27 ÷ ☐ = 3, students should use knowledge of the related multiplication fact of 3 × 9 = 27. They should ask themselves questions such as, "How many 3s are in 27 ?" or "3 times what number is 27?" Have them justify their thinking with models or drawings.

Instructional Resources/Tools

Sets of counters
Number lines to skip count and relate to multiplication
Arrays

Table 2. Common multiplication and division situations (Common Core State Standards for Mathematics 2010)

ORC # 4345 From the National Council of Teachers of Mathematics, Illuminations: Exploring equal sets.
This four-part lesson encourages students to explore models for multiplication, the inverse of multiplication, and representing multiplication facts in equation form.

ORC # 4343 From the National Council of Teachers of Mathematics, Illuminations: All About Multiplication
In this four-lesson unit, students explore several meanings and representation of multiplications and learn about properties of operations for multiplication.

The Math Bookshelf on the Ohio Resource Center site at http://ohiorc.org.

National Library of Virtual Manipulatives
The National Library of Virtual Manipulatives contains Java applets and activities for K-12 mathematics.
- Number Line Arithmetic: Illustrates arithmetic operations using a number line.
- Number Line Bars: Use bars to show addition, subtraction, multiplication, and division on a number line.

Common Misconceptions

Students think a symbol (? or []) is always the place for the answer. This is especially true when the problem is written as 15 ÷ 3 =? or 15 = ☐ x 3.

Students also think that 3 ÷ 15 = 5 and 15 ÷ 3 = 5 are the same equations. The use of models is essential in helping students eliminate this understanding.

The use of a symbol to represent a number once cannot be used to represent another number in a different problem/situation. Presenting students with multiple situations in which they select the symbol and explain what it represents will counter this misconception.

Diverse Learners

Information and instructional strategies for gifted students, English Language Learners (ELL), and students with disabilities is available in the Introduction to Universal Design for Learning document located on the Revised Academic Content Standards and Model Curriculum Development Web page. Additional strategies and resources based on the Universal Design for Learning principles can be found at www.cast.org.

Specific strategies for mathematics may include:
Allow students to use repeated addition for multiplication and repeated subtraction for division to prove/validate their answers.

Connections:

This cluster is connected to the Third Grade Critical Area of Focus #1, **Developing understanding of multiplication and division and strategies for multiplication and division within 100**. More informaiton about this critical area of focus can be found by clicking here.

Connect this domain with understanding properties of multiplication and the relationship between multiplication and division. (Grade 3 OA 5 – 6)

The use of a symbol for an unknown is foundational for letter variables in Grade 4 when representing problems using equations with a letter standing for the unknown quantity (Grade 4 OA 2 and OA 3).

Domain	Operations and Algebraic Thinking
Cluster	***Understand properties of multiplication and the relationship between multiplication and division.***
Standards	5. Apply properties of operations as strategies to multiply and divide. *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)* 6. Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

As the framework for the assessments, this section will be developed by the CCSS assessment consortia (SBAC and PARCC). Ohio is currently participating in both consortia and has input into the development of the frameworks. This information will be included as it is developed.

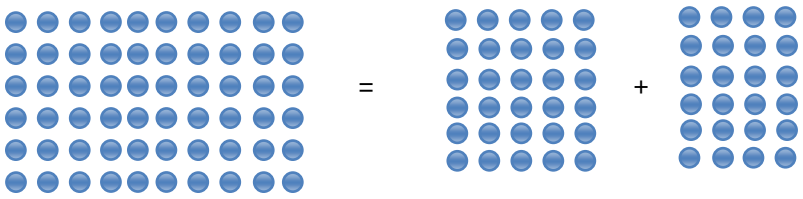
Instructional Strategies and Resources

Instructional Strategies

Students need to apply properties of operations (commutative, associative and distributive) as strategies to multiply and divide. Applying the concept involved is more important than students knowing the name of the property. Understanding the commutative property of multiplication is developed through the use of models as basic multiplication facts are learned. For example, the result of multiplying 3 x 5 (15) is the same as the result of multiplying 5 x 3 (15).

To find the product of three numbers, students can use what they know about the product of two of the factors and multiply this by the third factor. For example, to multiply 5 x 7 x 2, students know that 5 x 2 is 10. Then, they can use mental math to find the product of 10 x 7 (70). Allow students to use their own strategies and share with the class when applying the associative property of multiplication.

Splitting arrays can help students understand the distributive property. They can use a known fact to learn other facts that may cause difficulty. For example, students can split a 6 x 9 array into 6 groups of 5 and 6 groups of 4; then, add the sums of the groups.



The 6 groups of 5 is 30 and the 6 groups of 4 is 24. Students can write 6 x 9 as 6 x 5 + 6 x 4.

Students' understanding of the part/whole relationships is critical in understanding the connection between multiplication and division.

Instructional Resources/Tools

Ohio Resource Center # 3998, From the National Council of Teachers of Mathematics, Illuminations:– Multiplication: It's in the Cards - Students skip-count and examine multiplication patterns. They also explore the commutative property of

multiplication.

Ohio Resource Center # 10564, From the National Council of Teachers of Mathematics, Illuminations:- Multiplication: It's in the Cards: Looking for Calculator Patterns - Students use a web-based calculator to create and compare counting patterns using the constant function feature of the calculator. Making connections between multiple representations of counting patterns reinforces students understanding of this important idea and helps them recall these patterns as multiplication facts. From a chart, students notice that multiplication is commutative.

Common Misconceptions

Diverse Learners

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Connections:

This cluster is connected to the Third Grade Critical Area of Focus #1, **Developing understanding of multiplcation and division and strategies for multiplication and division within 100**. More information about this critical area of focus can be found by clicking here.

Grade 3

Domain	Operations and Algebraic Thinking
Cluster	***Multiply and divide within 100.***
Standards	7. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

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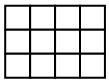
Instructional Strategies and Resources

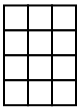
Instructional Strategies

Students need to understand the part/whole relationships in order to understand the connection between multiplication and division. They need to develop efficient strategies that lead to the big ideas of multiplication and division. These big ideas include understanding the properties of operations, such as the commutative and associative properties of multiplication and the distributive property. The naming of the property is not necessary at this stage of learning.

In Grade 2, students found the total number of objects using rectangular arrays, such as a 5 x 5, and wrote equations to represent the sum. This is called unitizing, and it requires students to count groups, not just objects. They see the whole as a number of groups of a number of objects. This strategy is a foundation for multiplication in that students should make a connection between repeated addition and multiplication.

As students create arrays for multiplication using objects or drawing on graph paper, they may discover that three groups of four and four groups of three yield the same results. They should observe that the arrays stay the same, although how they are viewed changes. Provide numerous situations for students to develop this understanding.





To develop an understanding of the distributive property, students need decompose the whole into groups. Arrays can be used to develop this understanding. To find the product of 3 × 9, students can decompose 9 into the sum of 4 and 5 and find 3 × (4 + 5).

The distributive property is the basis for the standard multiplication algorithm that students can use to fluently multiply multi-digit whole numbers in Grade 5.

Once students have an understanding of multiplication using efficient strategies, they should make the connection to division.

Using various strategies to solve different contextual problems that use the same two one-digit whole numbers requiring multiplication allows for students to commit to memory all products of two one-digit numbers.

Instructional Resources/Tools

Unifix cubes or cubes
Grid or graph paper
Sets of counters

Common Misconceptions

Diverse Learners

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Connections:

This cluster is connected to the Third Grade Critical Area of Focus #1, **Developing understanding of multiplcation and division and strategies for multiplication and division within 100**. More information about this critical area of focus can be found by clicking here.

Grade 3

Domain	Operations and Algebraic Thinking
Cluster	***Solve problems involving the four operations, and identify and explain patterns in arithmetic.***
Standards	8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. 9. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends*

Content Elaborations (in development)

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Expectations for Learning (in development)

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Instructional Strategies and Resources

Instructional Strategies

Students gain a full understanding of which operation to use in any given situation through contextual problems. Number skills and concepts are developed as students solve problems. Problems should be presented on a regular basis as students work with numbers and computations.

Researchers and mathematics educators advise against providing “key words” for students to look for in problem situations because they can be misleading. Students should use various strategies to solve problems. Students should analyze the structure of the problem to make sense of it. They should think through the problem and the meaning of the answer before attempting to solve it.

Encourage students to represent the problem situation in a drawing or with counters or blocks. Students should determine the reasonableness of the solution to all problems using mental computations and estimation strategies.

Students can use base–ten blocks on centimeter grid paper to construct rectangular arrays to represent problems.

Students are to identify arithmetic patterns and explain them using properties of operations. They can explore patterns by determining likenesses, differences and changes. Use patterns in addition and multiplication tables.

Instructional Resources/Tools

From the National Council of Teachers of Mathematics, Illuminations: Times. Students can also look for patterns in the table.

Ohio Resource Center # 3998, From the National Council of Teachers of Mathematics, Illuminations:– Multiplication: It’s in the Cards - Students skip-count and examine multiplication patterns. They also explore the commutative property of multiplication.

Ohio Resource Center # 10564, From the National Council of Teachers of Mathematics, Illuminations:– Multiplication: It's in the Cards: Looking for Calculator Patterns - Students use a web-based calculator to create and compare counting patterns using the constant function feature of the calculator. Making connections between multiple representations of counting patterns reinforces students understanding of this important idea and helps them recall these patterns as multiplication facts. From a chart, students notice that multiplication is commutative.

Common Misconceptions

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Connections:

This cluster is connected to the Third Grade Critical Area of Focus #1, **Developing understanding of multiplcation and division and strategies for multiplication and division within 100**. More information about this critical area of focus can be found by clicking here.

Represent and solve problems involving multiplication and division. (Grade 3 OA 1 – 4)

Use place value understanding and properties of operations to perform multi-digit arithmetic. (Grade 3 NBT 1 -3)

Grade 3

Domain	Number and Operations in Base Ten
Cluster	***Use place value understanding and properties of operations to perform multi-digit arithmetic.***
Standards	1. Use place value understanding to round whole numbers to the nearest 10 or 100. 2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. 3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

Content Elaborations (in development)

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Expectations for Learning (in development)

As the framework for the assessments, this section will be developed by the CCSS assessment consortia (SBAC and PARCC). Ohio is currently participating in both consortia and has input into the development of the frameworks. This information will be included as it is developed.

Instructional Strategies and Resources

Instructional Strategies

Prior to implementing rules for rounding students need to have opportunities to investigate place value. A strong understanding of place value is essential for the developed number sense and the subsequent work that involves rounding numbers.

Building on previous understandings of the place value of digits in multi-digit numbers, place value is used to round whole numbers. Dependence on learning rules can be eliminated with strategies such as the use of a number line to determine which multiple of 10 or of100, a number is nearest (5 or more rounds up, less than 5 rounds down). As students' understanding of place value increases, the strategies for rounding are valuable for estimating, justifying and predicting the reasonableness of solutions in problem-solving.

Strategies used to add and subtract two-digit numbers are now applied to fluently add and subtract whole numbers within 1000. These strategies should be discussed so that students can make comparisons and move toward efficient methods.

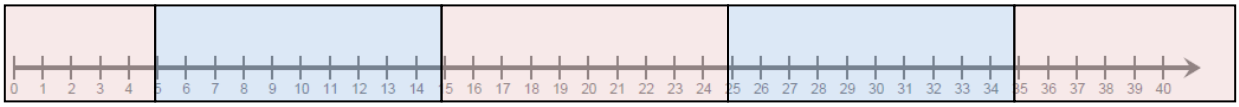
Number sense and computational understanding is built on a firm understanding of place value.

Understanding what each number in a multiplication expression represents is important. Multiplication problems need to be modeled with pictures, diagrams or concrete materials to help students understand what the factors and products represent. The effect of multiplying numbers needs to be examined and understood.

The use of area models is important in understanding the properties of operations of multiplication and the relationship of the factors and its product. Composing and decomposing area models is useful in the development and understanding of the distributive property in multiplication.

Continue to use manipulative like hundreds charts and place-value charts. Have students use a number line or a roller coaster example to block off the numbers in different colors.

For example this chart show what numbers will round to the tens place.



Rounding can be expanded by having students identify all the numbers that will round to 30 or round to 200.

Instructional Resources/Tools

Number lines
100s chart
National Library of Virtual Manipulatives

Rectangle Multiplication – Visualize the multiplication of two numbers as area.

ORC # 5793 From the Mathematics TEKS Toolkit, Make a hundred - Students roll a die seven times, each time determining whether to add that number of tens or that number of ones to make a sum as close to 100 as possible without going over.

Common Misconceptions

The use of terms like "round up" and "round down" confuses many students. For example, the number 37 would round to 40 or they say it "rounds up". The digit in the tens place is changed from 3 to 4 (rounds up). This misconception is what causes the problem when applied to rounding down. The number 32 should be rounded (down) to 30, but using the logic mentioned for rounding up, some students may look at the digit in the tens place and take it to the previous number, resulting in the incorrect value of 20. To remedy this misconception, students need to use a number line to visualize the placement of the number and/or ask questions such as: "What tens are 32 between and which one is it closer to?" Developing the understanding of what the answer choices are before rounding can alleviate much of the misconception and confusion related to rounding.

Diverse Learners

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Connections:

This cluster is connected to the Third Grade Critical Area of Focus #1**, Developing understanding of multiplcation and division and strategies for multiplication and division within 100**. Additionally, the content in this cluster goes beyond the critical areas to address solving multi-step problems. More information about this critical area of focus can be found by clicking here.

The rounding strategies developed in third grade will be expanded in grade four with larger numbers.

Additionally, students will formalize the rules for rounding numbers with the expansion of numbers in fourth grade.

In fourth grade the place value concepts developed in grades K-3 will be expanded to include decimal notaiton. Understand place value. (Grade 2 NBT 1 – 4 and Grade 2 NBT 5 – 9)

Domain	Number and Operations - Fractions
Cluster	***Develop understanding of fractions as numbers.***
Standards	1. Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*. 2. Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line. b. Represent a fraction *a*/*b* on a number line diagram by marking off *a* lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line. 3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model. c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.* d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

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Instructional Strategies and Resources

Instructional Strategies

This is the initial experience students will have with fractions and is best done over time. Students need many opportunities to discuss fractional parts using concrete models to develop familiarity and understanding of fractions. Expectations in this domain are limited to fractions with denominators 2, 3, 4, 6 and 8.

Understanding that a fraction is a quantity formed by part of a whole is essential to number sense with fractions. Fractional parts are the building blocks for all fraction concepts. Students need to relate dividing a shape into equal parts and representing this relationship on a number line, where the equal parts are between two whole numbers. Help students plot fractions on a number line, by using the meaning of the fraction. For example, to plot $4/5$ on a number line, there are 5 equal parts with 4 copies of the 5 equal parts.

5 equal parts make the whole

4 copies of the 5 equal parts represent the fractional amount

As students counted with whole numbers, they should also count with fractions. Counting equal-sized parts helps students determine the number of parts it takes to make a whole and recognize fractions that are equivalent to whole numbers.

Students need to know how big a particular fraction is and can easily recognize which of two fractions is larger. The fractions must refer to parts of the same whole. Benchmarks such as $1/2$ and 1 are also useful in comparing fractions.

Equivalent fractions can be recognized and generated using fraction models. Students should use different models and decide when to use a particular model. Make transparencies to show how equivalent fractions measure up on the number line.

Venn diagrams are useful in helping students organize and compare fractions to determine the relative size of the fractions, such as more than $1/2$, exactly $1/2$ or less than $1/2$. Fraction bars showing the same sized whole can also be used as models to compare fractions. Students are to write the results of the comparisons with the symbols >, =, or <, and justify the conclusions with a model.

Instructional Resources/Tools

Region or area models
Length or measurement models
Grid or dot paper (draw pictures to explore fraction ideas)
Set models
Geoboards
Fraction bars or strips

National Library of Virtual Manipulatives
- Fractions – Naming – Write the fraction corresponding to the highlighted portion of a shape.
- Fractions – Visualizing – Illustrate a fraction by dividing a shape and highlighting the appropriate parts.
- Fractions – Parts of a Whole – Relates parts of a whole unit to written description and fraction.

From the National Council of Teachers of Mathematics, Illuminations: Fun with Fractions: Investigating Equivalent Fractions with Relationship Rods - Students investigate the length model by working with relationship rods to find equivalent fractions. Students develop skills in reasoning and problem solving as they explain how two fractions are equivalent (the same length).

Transparencies you can make to show students how equivalent fractions measure up on the number line.
http://mathforum.org/paths/fractions/seeing.equiv.html

From the National Council of Teachers of Mathematics, Illuminations: Fun with Fractions - In this unit, students explore relationships among fractions through work with the length model. This early work with fraction relationships helps students make sense of basic fraction concepts and facilitates work with comparing and ordering fractions and working with equivalency.

Learn Fractions with Cuisenaire Rods
- template to create Cuisenaire rods
- equivalent fractions

Common Misconceptions

The idea that the smaller the denominator, the smaller the piece or part of the set, or the larger the denominator, the larger the piece or part of the set, is based on the comparison that in whole numbers, the smaller a number, the less it is, or the larger a number, the more it is. The use of different models, such as fraction bars and number lines, allows students to compare unit fractions to reason about their sizes.

Students think all shapes can be divided the same way. Present shapes other than circles, squares or rectangles to prevent students from overgeneralizing that all shapes can be divided the same way. For example, have students fold a triangle into eighths. Provide oral directions for folding the triangle:

1. Fold the triangle into half by folding the left vertex (at the base of the triangle) over to meet the right vertex.
2. Fold in this manner two more times.
3. Have students label each eighth using fractional notation. Then, have students count the fractional parts in the triangle (one-eighth, two-eighths, three-eighths, and so on).

Diverse Learners

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Connections:

This cluster is connected to the Third Grade Critical Area of Focus #2, Developing understanding of fractions, especially unit fractions(fractions with numerator 1). More information about this critical area of focus can be found by clicking here.

Partitioning traditional shapes into equal parts. (Grade 1 G 3)

Grade 3

Domain	Measurement and Data
Cluster	***Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.***
Standards	1. Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, *e.g., by representing the problem on a number line diagram.* 2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.

Content Elaborations (in development)

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Expectations for Learning (in development)

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Instructional Strategies and Resources

Instructional Strategies

A clock is a common instrument for measuring time. Learning to tell time has much to do with learning to read a dial-type instrument and little with time measurement.

Students have experience in telling and writing time from analog and digital clocks to the hour and half hour in Grade 1 and to the nearest five minutes, using a.m. and p.m. in Grade 2. Now students will tell and write time to the nearest minute and measure time intervals in minutes.

Provide analog clocks that allow students to move the minute hand.

Students need experience representing time from a digital clock to an analog clock and vice versa.

Provide word problems involving addition and subtraction of time intervals in minutes. Have students represent the problem on a number line. Student should relate using the number line with subtraction from Grade 2.

Provide opportunities for students to use appropriate tools to measure and estimate liquid volumes in liters only and masses of objects in grams and kilograms. Students need practice in reading the scales on measuring tools since the markings may not always be in intervals of one. The scales may be marked in intervals of two, five or ten.

Allow students to hold gram and kilogram weights in their hand to use as a benchmark. Use water colored with food coloring so that the water can be seen in a beaker.

Students should estimate volumes and masses before actually finding the measuring. Show students a group containing the same kind of objects. Then, show them one of the objects and tell them its weight. Fill a container with more objects and ask students to estimate the weight of the objects.

Use similar strategies with liquid measures. Be sure that students have opportunities to pour liquids into different size containers to see how much liquid will be in certain whole liters. Show students containers and ask, “How many liters do you think will fill the container?”

Instructional Resources/Tools

Beakers with whole number measures
Graduated cylinders
Measuring cups with liter markings

Balance scales
Pan or bucket balances
Weights in grams and kilograms
Objects to weigh
Food coloring
Water

Ohio Resource Center # 4021 - Teaching Clock This site has interactive clocks, games, quizzes, worksheets, and reference materials, all related to time. Analog and digital clocks help students in grades K-3 tell time to the nearest hour, half hour, 5 minutes, and 1 minute.

Virtual Manipulative Library

- Time – Analog and Digital Clocks - Interactively set the time on a digital and analog clock.
- Time - Match Clocks – Answer questions asking you to show a given time on digital and analog clocks.
- Time - What Time Will It Be? – Answer questions asking you to indicate what time it will be before or after a given time period.

http://www.pbs.org/teachers/mathline/lessonplans/esmp/ittakesten/ittakesten_procedure.shtm - Students estimate and measure marbles to the nearest gram and squeeze water-saturated sponges to practice measuring in milliliters.

Common Misconceptions

Students may read the mark on a scale that is below a designated number on the scale as if it was the next number. For example, a mark that is one mark below 80 grams may be read as 81 grams. Students realize it is one away from 80, but do not think of it as 79 grams.

Diverse Learners

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Connections:

This cluster goes beyond the Third Grade Critical Areas of Focus, **Solving multi-step problems**. More information about this critical area of focus can be found by clicking here.

Work with time and money. Grade 2 MD 7

Grade 3

Domain	Measurement and Data
Cluster	***Represent and interpret data.***
Standards	3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent pets.* 4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

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Instructional Strategies and Resources

Instructional Strategies

Representation of a data set is extended from picture graphs and bar graphs with single-unit scales to scaled picture graphs and scaled bar graphs. Intervals for the graphs should relate to multiplication and division with 100 (product is 100 or less and numbers used in division are 100 or less). In picture graphs, use values for the icons in which students are having difficulty with multiplication facts. For example, ☺ represents 7 people. If there are three ☺, students should use known facts to determine that the three icons represents 21 people. The intervals on the vertical scale in bar graphs should not exceed 100.

Students are to draw picture graphs in which a symbol or picture represents more than one object. Bar graphs are drawn with intervals greater than one. Ask questions that require students to compare quantities and use mathematical concepts and skills. Use symbols on picture graphs that student can easily represent half of, or know how many half of the symbol represents.

Students are to measure lengths using rulers marked with halves and fourths of an inch and record the data on a line plot. The horizontal scale of the line plot is marked off in whole numbers, halves or fourths. Students can create rulers with appropriate markings and use the ruler to create the line plots.

Instructional Resources/Tools

From the National Council of Teachers of Mathematics, Illuminations: Bar Grapher

This is a NCTM site that contains a bar graph tool to create bar graphs.

From the National Council of Teachers of Mathematics, Illuminations: All About Multiplication – Exploring equal sets
Students listen to the counting story, *What Comes in 2's, 3's, & 4's*, and then use counters to set up multiple sets of equal size. They fill in a table listing the number of sets, the number of objects in each set, and the total number in all. They study the table to find examples of the order (commutative) property. Finally, they apply the equal sets model of multiplication by creating pictographs in which each icon represents several data points.

From the National Council of Teachers of Mathematics, Illuminations: What's in a Name? – Creating Pictographs.
Students create pictographs and answer questions about the data set.

Common Misconceptions

Although intervals on a bar graph are not in single units, students count each square as one. To avoid this error, have students include tick marks between each interval. Students should begin each scale with 0. They should think of skip-

counting when determining the value of a bar since the scale is not in single units.

Diverse Learners

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Connections:

This cluster is connected to the Third Grade Critical Areas of Focus #2, **Developing understanding of fractions, especially unit fractions(fractions with numerator 1)** and goes beyond to address **Solving multi-step problems**. More information about this critical area of focus can be found by clicking here.

Represent and solve problems involving multiplication and division. (Grade 3 OA 1 – 4)

Multiply and divide within 100. (Grade 3 OA 7)

Solve problems involving the four operations, and identify and explain patterns in arithmetic. (Grade 3 OA 8 – 9)

Represent and interpret data. (Grade 2 MD 9 – 10)

Grade 3

Domain	Measurement and Data
Cluster	***Geometric measurement: understand concepts of area and relate area to multiplication and to addition.***
Standards	5. Recognize area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units. 6. Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units). 7. Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. b. Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths *a* and *b* + *c* is the sum of *a* × *b* and *a* × *c*. Use area models to represent the distributive property in mathematical reasoning. d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

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Instructional Strategies and Resources

Instructional Strategies

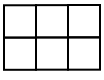
Students can cover rectangular shapes with tiles and count the number of units (tiles) to begin developing the idea that area is a measure of covering. Area describes the size of an object that is two-dimensional. The formulas should not be introduced before students discover the meaning of area.

The area of a rectangle can be determined by having students lay out unit squares and count how many square units it takes to completely cover the rectangle completely without overlaps or gaps. Students need to develop the meaning for computing the area of a rectangle. A connection needs to be made between the number of squares it takes to cover the rectangle and the dimensions of the rectangle. Ask questions such as:

- What does the length of a rectangle describe about the squares covering it?
- What does the width of a rectangle describe about the squares covering it?

The concept of multiplication can be related to the area of rectangles using arrays. Students need to discover that the length of one dimension of a rectangle tells how many squares are in each row of an array and the length of the other dimension of the rectangle tells how many squares are in each column. Ask questions about the dimensions if students do not make these discoveries. For example:

- How do the squares covering a rectangle compare to an array?
- How is multiplication used to count the number of objects in an array?



Students should also make the connection of the area of a rectangle to the area model used to represent multiplication.

This connection justifies the formula for the area of a rectangle.

Provide students with the area of a rectangle (i.e., 42 square inches) and have them determine possible lengths and widths of the rectangle. Expect different lengths and widths such as, 6 inches by 7 inches or 3 inches by 14 inches.

Instructional Resources/Tools

Square tiles

Rectangle Multiplication – Visualize the multiplication of two numbers as an area. This application allows student to create different size arrays and relate the array to the multiplication problem.

Common Misconceptions

Students may confuse perimeter and area when they measure the sides of a rectangle and then multiply. They think the attribute they find is length, which is perimeter. Pose problems situations that require students to explain whether they are to find the perimeter or area.

Diverse Learners

Information and instructional strategies for gifted students, English Language Learners (ELL), and students with disabilities is available in the Introduction to Universal Design for Learning document located on the Revised Academic Content Standards and Model Curriculum Development Web page. Additional strategies and resources based on the Universal Design for Learning principles can be found at www.cast.org.

Connections:

This cluster is connected to the Third Grade Critical Area of Focus #3, **Developing understanding of the structure of rectangular arrays and of area.** More information about this critical area of focus can be found by clicking here.

Fluently multiply and divide within 100 (3.OA.2.7).

Distributive property

Grade 3

Domain	Measurement and Data
Cluster	***Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.***
Standards	8. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

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Instructional Strategies and Resources

Instructional Strategies

Students have created rectangles before when finding the area of rectangles and connecting them to using arrays in the multiplication of whole numbers. To explore finding the perimeter of a rectangle, have students use nonstretchy string. They should measure the string and create a rectangle before cutting it into four pieces. Then, have students use four pieces of the nonstretchy string to make a rectangle. Two pieces of the string should be of the same length and the other two pieces should have a different length that is the same. Students should be able to make the connection that perimeter is the total distance around the rectangle.

Geoboards can be used to find the perimeter of rectangles also. Provide students with different perimeters and have them create the rectangles on the geoboards. Have students share their rectangles with the class. Have discussions about how different rectangles can have the same perimeter with different side lengths.

Students experienced measuring lengths of inches and centimeters in Grade 2. They have also related addition to length and writing equations with a symbol for the unknown to represent a problem.

Once students know how to find the perimeter of a rectangle, they can find the perimeter of rectangular-shaped objects in their environment. They can use appropriate measuring tools to find lengths of rectangular-shaped objects in the classroom. Present problems situations involving perimeter, such as finding the amount of fencing needed to enclose a rectangular shaped park, or how much ribbon is needed to decorate the edges of a picture frame. Also present problem situations in which the perimeter and two or three of the side lengths are known, requiring students to find the unknown side length.

Students need to know when a problem situation requires them to know that the solution relates to the perimeter or the area. They should have experience with understanding area concepts when they recognize it as an attribute of plane figures. They also discovered that when plane figures are covered without gaps by *n* unit squares, the area of the figure is *n* square units.

Students need to explore how measurements are affected when one attribute to be measured is held constant and the other is changed. Using square tiles, students can discover that the area of rectangles may be the same, but the perimeter of the rectangles varies. Geoboards can also be used to explore this same concept.

Instructional Resources/Tools

Square tiles
1-inch or 1-centimeter grid paper
Nonstretchy string
Geoboards and rubber bands

From PBS Teacher – For Real: Penned In - Explore rectangles and perimeter in real-world applications. In this video

clip from Cyberchase, Harry builds a rectangular fence with an assortment of different-size sections but forgets to add a gate to get out.

National Library of Virtual Manipulatives - Making a 5 Peg Triangle - Use geoboards to illustrate area, perimeter, and rational number concepts.

Ohio Resource Center # 11115 Spaghetti and Meatballs for All!: A Mathematical Story - - An entry on the Ohio Resource Center *Mathematics Bookshelf,* this book provides students with a real-life context for investigating variation in perimeter while area remains constant. In the story, small tables are pushed together to make one large table, until too many people show up, and the large table has to be subdivided into smaller arrangements to provide more seating. Activities and extensions are suggested at the back of the book.

From the National Council of Teachers of Mathematics, Illuminations: Junior Architects - Finding Perimeter and Area In this lesson, students develop strategies for finding the perimeter and area for rectangles and triangles using geoboards and graph paper. Students learn to appreciate how measurement is a critical component to planning their clubhouse design.

Common Misconceptions

Students think that when they are presented with a drawing of a rectangle with only two of the side lengths shown or a problem situation with only two of the side lengths provided, these are the only dimensions they should add to find the perimeter. Encourage students to include the appropriate dimensions on the other sides of the rectangle. With problem situations, encourage students to make a drawing to represent the situation in order to find the perimeter.

Diverse Learners

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Connections:

This cluster is connected to the Third Grade Critical Area of Focus #3, **Developing understanding of the structure of rectangular arrays and of area**. More information about this critical area of focus can be found by clicking here.

Measure and estimate lengths in standard units. Grade 2 MD 1 – 4

Relate addition and subtraction to length. Grade 2 MD 5 – 6

Grade 3

Domain	**Geometry**
Cluster	***Reason with shapes and their attributes.***
Standards	1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. 2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

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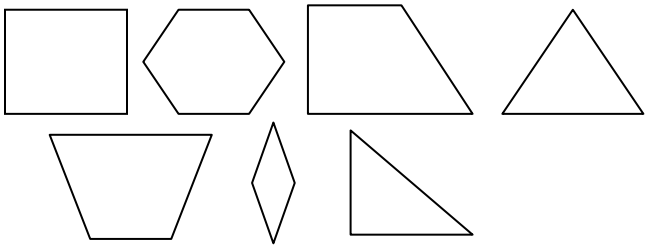
Instructional Strategies and Resources

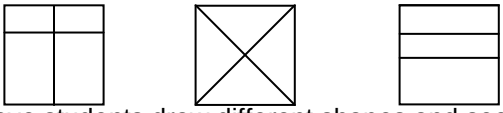
Instructional Strategies

In earlier grades, students have experiences with informal reasoning about particular shapes through sorting and classifying using their geometric attributes. Students have built and drawn shapes given the number of faces, number of angles and number of sides.

The focus now is on identifying and describing properties of two-dimensional shapes in more precise ways using properties that are shared rather than the appearances of individual shapes. These properties allow for generalizations of all shapes that fit a particular classification. Development in focusing on the identification and description of shapes' properties should include examples and nonexamples, as well as examples and nonexamples drawn by students of shapes in a particular category. For example, students could start with identifying shapes with right angles. An explanation as to why the remaining shapes do not fit this category should be discussed. Students should determine common characteristics of the remaining shapes.



In Grade 2, students partitioned rectangles into two, three or four equal shares, recognizing that the equal shares need not have the same shape. They described the shares using words such as, halves, thirds, half of, a third of, etc., and described the whole as two halves, three thirds or four fourths. In Grade 4, students will partition shapes into parts with equal areas (the spaces in the whole of the shape). These equal areas need to be expressed as unit fractions of the whole shape, i.e., describe each part of a shape partitioned into four parts as $\mathbf{1/4}$ of the area of the shape.



Have students draw different shapes and see how many ways they can partition the shapes into parts with equal area.

Resources/Tools

From the National Council of Teachers of Mathematics, Illuminations: Rectangles and Parallelograms - While exploring properties of rectangles and parallelograms using dynamic software, students identify, compare, and analyze attributes of both shapes through physical and mental manipulation.

Exploring Properties of Rectangles and Parallelograms Using Dynamic Software **-** Dynamic geometry software provides an environment in which students can explore geometric relationships and make and test conjectures. In this example, properties of rectangles and parallelograms are examined. The emphasis is on identifying what distinguishes a rectangle from a more general parallelogram

Misconceptions

Students may identify a square as a "nonrectangle" or a "nonrhombus" based on limited images they see. They do not recognize that a square is a rectangle because it has all of the properties of a rectangle. They may list properties of each shape separately, but not see the interrelationships between the shapes. For example, students do not look at the properties of a square that are characteristic of other figures as well. Using straws to make four congruent figures have students change the angles to see the relationships between a rhombus and a square. As students develop definitions for these shapes, relationships between the properties will be understood.

Diverse Learners

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Students can increase their understanding of the relationships of shapes through specific approaches to sorting shapes, such as using a Venn diagram to place shapes with four congruent sides in one circle and those with a right angle in another circle. This should lead students to realize that the two circles must overlap with the squares in the overlapping region.

Connections:

This cluster is connected to the Third Grade Critical Areas of Focus #3**, Developing understanding of the structure of rectangular arrays and of area** and #4, **Describing and analyzing two-dimensional shapes**. More information about this critial area of focus can be found by clicking here.

Reason with shapes and their attributes. (Grade 2 G 3)